temperature $T$; p, partial pressure of water vapor in gas; $\alpha ; \beta$, coefficients of heat and mass transfer; $\theta$, gas temperature; $C_{M}(W)$, enthalpy of material at moisture content $W$; $\rho_{M}(W)$, density of material at moisture content $W$; $\xi$, drag coefficient of medium; $\rho(\theta$, $p$ ), gas density at temperature $\theta$ and water-vapor partial pressure $p ; v_{g}$, gas velocity; $g$, acceleration due to gravity; d, moisture content of gas; $\mathrm{r}_{\mathrm{T}}$, specific heat of vaporization at temperature T ; $\Lambda$, circumference of drying column; $K$, coefficient of heat transfer from gas through wall to surrounding medium at temperature $T_{\text {ext }}$; $S$, cross-sectional area of drying column.

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## KINETICS OF THE SOLUTION OF POLYOXYETHYIENE IN WATER

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The kinetics of the process of solution of WSR-301 polyoxyethylene in water is investigated using a quadratic Doppler spectrometer.

It is known that several weak polymer solutions have anomalously low drag in turbulent motion along tubes. The most pronounced effect is noted for the synthetic polymer polyoxyethylene (PEO), the hydrodynamic action of which depends on the molecular weight, concentration, and conditions of technical preparation of the sample. It is of interest to study these properties, so as to determine the optimal parameters for efficiency of PEO as $a$ dragreducing agent.

The present investigation is devoted to WSR-301 PEO, which has the following characteristics. For velocity gradient $g=0$, the characteristic viscosity, measured on a Zimm rotational viscometer, is $[\eta]_{g=0}=6.5 \mathrm{~d} / / \mathrm{g}$. The Mark-Kuhn-Khauvink formula $[\eta]_{\text {water }}^{2 s}=1.25$. $10^{4} \cdot \mathrm{M}^{\circ} \cdot{ }^{78}$ gives a value of $1 \cdot 10^{6}$ for the molecular weight. The maximum reduction in drag is observed approximately 2 h after (as shown by visual observation and decanting the waterpolymer mixture) solution of PEO in water is complete. If the solution is kept for some time, a sharp drop in its efficiency is observed within the first day [1]. The aim of the present work is to explain these changes in the drag-reducing properties of PEO.

It seems likely that change in the efficiency of the polymer with time is associated with changes in the structure of the macromolecules in the solution or with the dynamics of the process of solution. A clearer idea can be obtained by studying the light-scattering properties of the solution over various time intervals from its moment of preparation. It is known that light scattered by moving particles undergoes a Doppler frequency shift. For suspended particles diffusing in liquids, this shift may vary from a fraction of a hertz to

[^0][^1]TABLE 1. Time Dependence of Size of Polymer Particles in Solution

| Holding time, <br> h | Half-width of <br> scattered radia- <br> tion $\Delta_{90^{\circ}}, \mathrm{Hz}$ | Diffusion coef- <br> ficient $\mathrm{D}, \mathrm{cm}^{2} /$ <br> sec | Diameter of <br> suspended par- <br> ticles $\mathrm{d}, \mu$ |
| ---: | ---: | :--- | :--- |
|  |  |  |  |
| 3 | 200 | $1,0 \cdot 10^{-8}$ | 0,45 |
| 9 | 50 | $0,25 \cdot 10^{-8}$ | 1,8 |
| 24 | 300 | $1,5 \cdot 10^{-8}$ | 0,3 |
| 30 | 450 | $2,2 \cdot 10^{-8}$ | 0,24 |
|  | 450 | $2,2 \cdot 10^{-8}$ | 0,24 |

several kilohertz. In the case of random Brownian motion of the particles, the values of the shift are distributed continuously over a frequency band determined by the diffusion coefficient for these particles. For a monodisperse system of particles, the spectrum of scattered light is broadened, taking the Lorentz form [2]

$$
f(v)=\frac{\Delta}{\Delta^{2}+\left(v-v_{0}\right)^{2}}
$$

and the half-width of the spectrum of scattered radiation is $\Delta=D K^{2}$. Here $D$ is the diffusion coefficient of Brownian particles, which is itself related to the particle radius $a$ and the viscosity $\eta$ of the medium by the Einstein formula
and

$$
D=\frac{k T}{6 \pi \eta a},
$$

$$
K^{2}=\left(2\left(\frac{2 \pi}{\lambda}\right) \sin \frac{\theta}{2}\right)^{2}
$$

where $\lambda$ is the wavelength of the radiation used ( $0.64 \mu$ ); $\theta$ is the scattering angle ( $90^{\circ}$ ); $v_{0}$ is the frequency of the incident radiation.

The spectral broadening was measured on a quadratic Doppler spectrometer [3]. The system investigated is polydisperse and, strictly speaking, these equations cannot be used. However, in view of the form of the spectrum, it can be assumed that at certain moments of time the contour is close to Lorentzian, which indicates that particles of the same size predominate in the solution.

Table 1 shows the results of analyzing the spectra of a solution of WSR-301 polyoxyethylene (concentration $c=10^{-3} \mathrm{~g} / \mathrm{cm}^{3}$ ) obtained after holding for $1,3,9,24$, and 30 h .

According to these data, the kinetics of solution of WSR-301 polyoxyethylene in water appears to be as follows. As the powdery polymer dissolves, it breaks down first of all not to individual macromolecules, but to structural units which have a relatively high stability in solution because of the particular features of the synthesis of the WSR-301 polymer. These structural units (of size $0.45 \mu$ ) consist of macromolecules pressed together in the process of synthesis. Decomposition into these structural units is complete after about 1 h. After 3 h the molecules composing these units swell and unfold in the aqueous solution. The units then increase considerably in size, and their translational mobility is reduced correspondingly, which is reflected in the spectrum of scattered light (Table l). After holding the solution for 9 h , the units begin to decompose into individual macromolecules of the polymer ( $0.24 \mu$ ) because of thermal motion. After about a day, these units are completely decomposed.

These results show that the maximum size of the macroparticles ( $1.8 \mu$ ) is observed 3 h after the moment of preparation of the solution. As this is also the time at which the maximum hydrodynamic effect is observed, it may be assumed that the drop in hydrodynamic efficiency of the solution of WSR-301 PEO in the course of the first day of holding is associated with the decrease in the percentage content by volume in the solution of particles of size $1.8 \mu$.

Similar investigations were carried out for a glycerine paste of PEO (1:1). The concentration of the solution of the paste in water was $c=10^{-3} \mathrm{~g} / \mathrm{cm}^{3}$. The size of the macroparticles was found to be a maximum ( $1.8 \mu$ ) after holding for 1.5 h . This indicates that the effect of the glycerine is to double the rate of swelling and unfolding of the stable structural units in solution.

Different companies use different technical procedures for preparing PEO. WSR-301 PEO is synthesized in the form of structural units, i.e., even in the dry state, this polymer has a complex aggregation of individual macromolecules in groups. This aggregation is evidently reflected in the size of the drag resistance produced in turbulent flow.

## NOTATION

g, velocity gradient, $\sec ^{-1}$; [ $\quad \mathrm{n}$ ], characteristic viscosity of solution, $\mathrm{d} 1 / \mathrm{g}$; M, molecular weight of polymer; $\Delta$, half-width of scattered-light spectrum, Hz ; $\nu_{0}$, frequency of incident radiation, Hz ; D , diffusion coefficient of Brownian particles, $\mathrm{cm}^{2} / \mathrm{sec}$; $k$, Boltzmann constant, erg/deg; $T$, absolute temperature, ${ }^{\circ} \mathrm{K} ; ~ \eta$, viscosity of solution, $p ; a$, particle radius, $\mu ; \mathrm{d}=2 \mathrm{kT} / 6 \pi n \mathrm{D}$, particle diameter, $\mu ; \mathrm{K}^{2}$, wave number; $\lambda$, wavelength of radiation used, $\mu ; 6$, scattering angle, deg; $c$, polymer concentration in solution, $g / \mathrm{cm}^{3}$.

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A. FINITE-DIFFERENCE METHOD FOR SOLVING INVERSE BOUNDARY-VALUE

PROBLEMS OF HEAT CONDUCTION
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UDC 536.24 .02

A finite-difference search method is described for determining the temperature and heat flux on one boundary of the body if the temperature and heat flux on the other boundary are known. The results of numerical experiments, which show that the method has proved to be efficient, are discussed.

Among the methods at our disposal for solving inverse boundary-value problems of heat conduction (IPHC) fully surveyed in [1-3] an important part is played by the variational methods. The latter are based on the minimization of a functional representing a discrepancy measured by using some norm. To solve IPHC in variational formulation one can use [3-9] the method of least squares, the search methods of the gradient type, or the trial-and-error method.

In the present article the method of finite differences is used to solve boundary IPHC based on the search for a temperature function which satisfies the heat-conduction equation and one of the boundary conditions over the entire region under consideration, the other boundary condition being satisfied in a countable set of points only.

An IPHC is considered for a plate ( $0 \leqslant x \leqslant L$ ) described by the following system of equations:

$$
\begin{align*}
c \rho \frac{\partial t}{\partial \tau}=\frac{\partial}{\partial x}\left(\lambda \frac{\partial t}{\partial x}\right) & ,(0<x<L, \tau>0),  \tag{I}\\
t(x, 0) & =\varphi(x),  \tag{2}\\
t(0, \tau) & =\psi(\tau)  \tag{3}\\
\frac{\partial t(0, \tau)}{\partial x} & =\psi_{1}(\tau) . \tag{4}
\end{align*}
$$

Institute of Thermophysical Engineering, Kiev. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 32, No. 3, pp. 502-507, March, 1977. Original article submitted February 10, 1976.

[^2]
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